Understanding and Applying Multilevel Models in Maternal and Child Health Epidemiology and Public Health

Adam C. Carle, M.A., Ph.D.
adam.carle@cchmc.org

Division of Health Policy and Clinical Effectiveness
James M. Anderson Center for Health Systems Excellence
Cincinnati Children’s Hospital Medical Center
University of Cincinnati School of Medicine
Cincinnati, OH

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Introduction

- Epidemiological research increasingly seeks to understand simultaneous influences of individual and contextual level variables.
  - Example 1:
    - Individual outcome: Asthma symptom severity.
    - Individual predictor: Presence of respiratory allergies.
      - Level 1
    - Contextual predictor: Neighborhood pollutant levels.
      - Level 2
Introduction

• Epidemiological research increasingly seeks to understand simultaneous influences of individual and contextual level variables.

  – Example 2:
    • Individual outcome: Health rating.
    • Individual predictor: Individual’s education.
      – Level 1
    • Contextual predictor: County unemployment rate.
      – Level 2
Introduction

• Intensified interest in variation within and across contexts.
  – What predicts variation across contexts?
    • Why do similar children living in different neighborhoods have disparate outcomes?
    • Why do comparable people living in different counties have heterogeneous outcomes?
  – What predicts variation within a context?
    • Why do children in the same neighborhood have dissimilar outcomes?
    • Why do people in the same county have diverse outcomes?
Introduction

• Multilevel models (MLM) offer a relatively new approach to understanding individual and contextual influences on health.

• MLM allow one to explicitly investigate sources of variation within and across contexts.
  – Across counties, do we see the same relationship between education and health?
  – Do we see variation in the relationship between a predictor and an outcome across contexts?
    • Requires thoughtful sampling.
Introduction

• Sampling designs can organize populations into clusters and collect data within clusters.
  – Example 1: Identify neighborhoods in a city’s area.
    • Randomly sample within each neighborhood.
      – Neighborhood = cluster.
  – Example 2: Identify counties in a state.
    • Randomly sample within each county.
      – County = cluster.
  – Examine cluster and individual level health influences.
Introduction

• Clustered designs result in non-independent data.
  – People within the clusters more similar to each other than to people in other clusters.

• Results in biased standard errors and parameters when analyzed using techniques that do not account for data’s clustered nature.
  – Increased Type I error.
    – (Chambers et al., 2003; Graubard et al., 1996).
Introduction

- Failing to address multilevel nature can lead to substantially biased results and inferences.

» Image courtesy of the Centre for Multilevel Modeling
Introduction

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Introduction

• Analysts traditionally treat clustered nature of complex/cluster sampling designs as a nuisance.
  – Adjust standard errors for sampling design.
    • Generalized estimating equations.
    • Complex survey methods.
  – Delivers correct standard errors, but....

• **Fails** to allow examinations of between-cluster variance unaccounted for by predictors.
  – (Merlo, et al., 2006).

• Often of interest in epidemiology.
Introduction

• MLM offer a solution.

• Account for data’s clustered nature *and* allow investigating sources of variation within and across clusters.

• How do MLM do this?
Introduction

• First, consider “typical” (OLS) regression.
  – Predict outcome from predictor set.
  – Ignores cluster membership.
  – One equation for entire sample.

• Essentially like fitting a regression in one cluster.
  – e.g., One county.
  \[
  Health_i = \beta_0 + \beta_1 Education_i + e_i
  \]
Introduction

• But, suppose we investigate multiple counties.

• MLM regression.
  – *Within each cluster*, predict outcome from predictor set.
  – One equation for each cluster (context).
    • *Within each cluster*, predict outcome from set.
    • Examine relationship between health and education for each county.
Introduction

• MLM regression.
  – MLMs “collect” the equations across clusters.
• Essentially give average relationship between health and education across counties.

• Describe variation in the size of the relationship between health and education across counties.
  – Variance component.
  – New feature of MLM relative to OLS regression.

• Examine covariation in size of the relationship between education and health across counties and counties’ averages.
  – Covariance component.
  – Another new feature.
Software

• Several programs available to fit MLMs.
  – No program will meet all your needs.
    • See resource list for references for each program.

• Today we’ll use MLwiN.
  – Graphical interface.
    • Command interface if desired.
  – Properly handles design weights.
    • See Carle (2009) for details.
  – Contextual or longitudinal designs.
  – Can do basic data manipulation.
**Example**

- Interpretative example.

- Fit a series of MLM examining whether:
  - Person’s education (education).
    - Individual variable.
      - Level 1
    - (and) County unemployment rate (unemployment).
      - Context variable.
      - Level 2
  - Predict individuals’ ratings of their general health.

- “Typical” set of models in a MLM analysis.
Methods

- Used data from the 2008 Ohio Family Health Survey (OFHS).
  - Individuals ($n = 50,830$) clustered within counties.
  - Stratified, list-assisted random digit dial survey.
    - Stratified by county.
    - Oversampled African Americans, Asian Americans, and people of Hispanic origin.
  - Represents non-institutionalized Ohio population.
Methods

• For simplicity’s sake:
  – Data clustered data by counties.
    • Does not precisely reflect sample design.
  – Uses unweighted data.
    • Weighted data **REQUIRE** special techniques.
      – See resources.
      – Carle (2009).
  – Uses complete cases only.
    • Subpopping **REQUIRES** special techniques.
      – See resources.
Methods

• MLMs examined individuals’ ratings of their health:
  – Health:
    1. Excellent.
    2. Very good.
    3. Good.
    4. Fair.
    5. Poor.
Methods

• Predicted as a function of:
  – Level-1 predictor:
    • Highest education completed.
      1. Less Than 1st Grade.
      2. First Through 8th Grade.
      3. Some High School, But No Diploma.
      4. High School Graduate Or Equivalent.
      5. Some College, But No Degree.
      6. Associate Degree.
      7. Four Year College Graduate.
      8. Advanced Degree.
  – Level-2 predictor:
    • County unemployment rate.
Models

- **Unconditional model.**
  - Examines whether average health ratings (health) varies across counties.

- **Level-1 predictor only.**
  - Does education predict health and does that relationship differ across counties?

- **Level-2 only predictor model.**
  - Does unemployment in a county (unemployment) affect health?
Models

- Model including level-1 and -2 predictors but no cross-level interaction.
  - Investigates contributions of level-1 and level-2 predictors simultaneously.

- Model including level-1 and -2 predictors and a cross-level interaction.
  - Asks whether relationship between education and health differs according to a county’s unemployment rate.

- All models allowed intercept (constant) to vary across counties.
Unconditional Model

• Unconditional model includes no predictors.
  – Examines whether average health ratings (health) varies across counties.
Results and Discussion

- Intercept term describes:
  - Average county-level health rating.
- $2.645 \ (p < 0.01)$.
Results and Discussion

• Two variance components describe:
  – Extent to which average health varies across counties.
    • $0.024 \ (p < 0.01)$.
  – Amount of residual variance within counties across individuals.
    • $1.205 \ (p < 0.01)$.

\[
\begin{align*}
\text{d30}_{ij} & \sim N(\mathbf{X}\beta, \Omega) \\
\beta_{0ij} & = \beta_{0y}\text{cons} \\
\beta_{0ij} & = 2.645(0.018) + u_{0j} + e_{0ij} \\
\begin{bmatrix} u_{0j} \end{bmatrix} & \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.024(0.004) \end{bmatrix} \\
\begin{bmatrix} e_{0ij} \end{bmatrix} & \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 1.205(0.008) \end{bmatrix}
\end{align*}
\]
Results and Discussion

• “Unconditional” model:

• Intraclass correlation describes:
  – Extent to which individuals in same county are similar to each other relative to individuals in different counties.
  – Proportion of total residual variance due to between group (county) differences.
  – Computed from the variance components.

\[
\frac{\text{Variance}_{\text{Between}}}{\text{Variance}_{\text{Between}} + \text{Variance}_{\text{Within}}} = 0.02(2\%) 
\]
Results and Discussion

• On average across counties, individuals rate their health a 2.645.

• Variance exists in this mean across counties (0.024).

• But, even more variance exists within counties (1.205).

\[ d_{30ij} \sim N(XB, \Omega) \]
\[ d_{30ij} = \beta_{0ij} \text{cons} \]
\[ \beta_{0ij} = 2.645(0.018) + u_{0j} + e_{0ij} \]
\[ [u_{0j}] \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.024(0.004) \end{bmatrix} \]
\[ [e_{0ij}] \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 1.205(0.008) \end{bmatrix} \]
Results and Discussion

• Model serves as a baseline comparison for more developed models.

• Fit this model to examine relative changes in parameters as one adds predictors.

• Can create pseudo $R^2$ statistic from relative changes.
  – Reduction in residual variance within contexts (counties) by adding predictors.
Level 1 Predictor

• Level 1 predictor with fixed and random effects:
  – Does education predict health?
  – Does the relationship between education and health differ across counties (contexts)?
  • Random effect.

• Before fitting this model, must consider Level 1 predictor’s scale/location.
Centering

• Variable “location” influences inferences and interpretation in single- and multilevel models.

• Level 1 intercept’s meaning depends on scale/location of Level 1 predictor variables.
  – See resource list for articles discussing these concepts in MLM.

• We will use group mean centering at Level 1 and grand mean centering at Level 2.
Results and Discussion

• Level-1 predictor with fixed and random effects:
  – Does education predict health?
  – Does the relationship between education and health differ across counties (contexts)?

\[
d30_{ij} \sim N(XB, \Omega)
\]
\[
d30_{ij} = \beta_{0ij} \text{cons} + \beta_{1ij} (\text{educ\_imp-m(county\_imp)})_{ij}
\]
\[
\beta_{0ij} = 2.645(0.018) + u_{0j} + e_{0ij}
\]
\[
\beta_{1ij} = -0.190(0.005) + u_{1j}
\]
\[
\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.025(0.004) \\ -0.004(0.001) & 0.001(0.000) \end{bmatrix}
\]
\[
\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 1.113(0.007) \end{bmatrix}
\]
Results and Discussion

- **Level-1 predictor fixed effect:**
  - Does education predict health?

- **Slope:** Relation between education and health.
  - \(-0.190\) (\(p < 0.01\)): As education decreases, health decreases.

\[
\begin{align*}
d_{30ij} &\sim N(XB, \Omega) \\
d_{30ij} = \beta_{0ij}\text{cons} + \beta_{1ij}(\text{educ}_\text{imp}-m(\text{county}_\text{imp}))_{ij} \\
\beta_{0ij} &= 2.645(0.018) + u_{0ij} + e_{0ij} \\
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\end{align*}
\]

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  u_{1ij}
\end{bmatrix}
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\begin{bmatrix}
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  1.113(0.007)
\end{bmatrix}
\]
Results and Discussion

• Level-1 predictor fixed effect:
  – Does education predict health?

• Variance in the intercepts between counties.
  – 0.025 ($p < 0.01$).

\[
\begin{align*}
d_{ij} & \sim N(XB, \Omega) \\
d_{ij} & = \beta_{0ij}\text{cons} + \beta_{1ij}(\text{educ}_\text{imp}-\text{m(county}_\text{imp}))_{ij} \\
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\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} & \sim N(0, \Omega_u) \quad \Omega_u = \begin{bmatrix} 0.025(0.004) \\ -0.004(0.001) & 0.001(0.000) \end{bmatrix} \\
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\end{align*}
\]
Results and Discussion

• Level-1 predictor fixed effect:
  – Does education predict health?

• Residual variance (1.113) now describes amount of variance within-counties after accounting for education’s influence on health.
Results and Discussion

• Create pseudo $R^2$ statistic from relative changes in within county residual variance.
  – Unconditional model = 1.205.
  – Level 1 predictor model = 1.113.
  – $R^2_e = \frac{(1.205 - 1.113)}{(1.205)} = .075$
  – 8% of variance within counties attributable to differences in education within counties.
Results and Discussion

• Two new random effects:

• Variance in slopes across counties.
  – Does relationship between individual predictor and outcome depend on context?
  – $0.001 (p < 0.01)$.

• Education-health relationship depends on county.

\[
\begin{align*}
d_{30ij} & \sim N(XB, \Omega) \\
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\end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix}1.113(0.007) \end{bmatrix}
\]
Results and Discussion

- **Two new random effects:**
- **Covariance between slope and counties’ intercepts.**
  - Describes whether effect of education on health varies as a function of counties’ means.
  - Negative: -0.004 ($p < 0.01$).

\[
\begin{align*}
\mathbf{d}_{30ij} &\sim N( XB, \Omega) \\
\mathbf{d}_{30ij} &= \beta_{0ij}\text{cons} + \beta_{1ij}(\text{educ}\_\text{imp}\_\text{m}(\text{county}\_\text{imp}))_{ij} \\
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1.113(0.007)
\end{bmatrix}
\end{align*}
\]
Results and Discussion

- **Two new random effects:**
  - Covariance between slope and county intercepts.
    - In counties where individuals rate their health more poorly on average, having less education has a more detrimental effect relative to counties where individuals rate their health better on average.

\[
\begin{align*}
d_{30 ij} & \sim N(X \beta, \Omega) \\
d_{30 ij} & = \beta_{0 ij} \text{cons} + \beta_{1 ij}(\text{educ \_imp - m(county \_imp)})_{ij} \\
\beta_{0 ij} & = 2.645(0.018) + u_{0 ij} + e_{0 ij} \\
\beta_{1 ij} & = -0.190(0.005) + u_{1 ij} \\
\begin{bmatrix} u_{0 ij} \\ u_{1 ij} \end{bmatrix} & \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.025(0.004) \\ -0.004(0.001) & 0.001(0.000) \end{bmatrix} \\
\begin{bmatrix} e_{0 ij} \end{bmatrix} & \sim N(0, \Omega_\varepsilon) : \Omega_\varepsilon = \begin{bmatrix} 1.113(0.007) \end{bmatrix}
\end{align*}
\]
Results and Discussion

- Two new random effects:
- Covariance between slope and county intercepts.
  - Can place covariance in correlation metric.

\[
\sigma_{01} = \frac{(-0.001)}{\sqrt{0.025 \times 0.001}} = -0.685
\]
Results and Discussion

- Level-2 predictor only model:
  - Does a county's unemployment rate predict individuals’ health?
    - 7.094 ($p < 0.01$).
    - Yes.

\[ d_{30, ij} \sim N(XB, \Omega) \]
\[ d_{30, ij} = \beta_{0ij} \text{cons} + 7.094 \times (1.132) \times (\text{unemrate-gm})_j \]
\[ \beta_{0ij} = 2.632(0.015) + u_{0j} + e_{0ij} \]
\[ [u_{0j}] \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.016 & 0.003 \\ 0.003 & 0.008 \end{bmatrix} \]
\[ [e_{0ij}] \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 1.205 & 0.008 \end{bmatrix} \]
Results and Discussion

• Residual variance (1.205) represents within county variance after controlling for unemployment rate.

\[
d30_{ij} \sim N(XB, \Omega)
d30_{ij} = \beta_{0ij} \text{cons} + 7.094(1.132)(\text{unemrate-gm})_j
\]

\[
\beta_{0ij} = 2.632(0.015) + u_{0j} + e_{0ij}
\]

\[
\begin{bmatrix} u_{0j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.016(0.003) \end{bmatrix}
\]

\[
\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 1.205(0.008) \end{bmatrix}
\]
Results and Discussion

- Variance in the intercept (.016) now describes variance in health after accounting for unemployment.

\[
d30_{ij} \sim N(XB, \Omega)
\]
\[
d30_{ij} = \beta_{0ij} \text{cons} + 7.094(1.132)(\text{unemrate-gm})_j
\]
\[
\beta_{0ij} = 2.632(0.015) + u_{0j} + e_{0ij}
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\[
[u_{0j}] \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.016 & (0.003) \\ (0.003) & 1.205(0.008) \end{bmatrix}
\]
Level-1 and Level-2 Model

• Model with Level 1 and Level 2 predictors.
  – Examine relationship between unemployment rate and health after controlling for within county education differences.

  – Does the relationship between health and education differ across counties after controlling for differences in unemployment rate?

• Answers whether contextual level variable predicts variance in the relationship between education and health across contexts.
Results and Discussion

- Interpret estimates in light of variables in model.

  - Intercept (2.634: $p < 0.01$) reflects estimated unadjusted average health after controlling for education.

\[
\begin{align*}
\text{d}_{30ij} & \sim N(XB, \Omega) \\
\text{d}_{30ij} & = \beta_{0ij}\text{cons} + \beta_{1ij}(\text{educ}_\text{imp}-m(\text{county}_\text{imp})){ij} + \\
& \quad + 6.081(1.019)(\text{unemrate}\text{-gm}){ij} \\
\beta_{0ij} & = 2.634(0.015) + u_{0ij} + e_{0ij} \\
\beta_{1ij} & = -0.190(0.005) + u_{1ij} \\
\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} & \sim N(0, \Omega_u) : \Omega_u = \\
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& \begin{bmatrix} 1.113(0.007) \end{bmatrix}
\end{align*}
\]
Results and Discussion

• Interpret estimates in light of variables in model.
  – Education slope (-0.19) shows that, even after accounting for unemployment, negative relationship between education and health.

\[
d_{30,ij} \sim N(XB, \Omega)
\]

\[
d_{30,ij} = \beta_{0ij} \text{cons} + \beta_{1j}(\text{educ}_\text{imp} - \text{m(county}_\text{imp}))_{ij} + 6.081(1.019)(\text{unemrate}_\text{gm})_{ij}
\]

\[
\beta_{0ij} = 2.634(0.015) + u_{0j} + e_{0ij}
\]

\[
\beta_{1j} = -0.190(0.005) + u_{1j}
\]

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\end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = 
\begin{bmatrix}
  1.113(0.007)
\end{bmatrix}
\]
Results and Discussion

• Interpret estimates in light of variables in model.
  – Similarly, individuals who live in counties with higher unemployment tend to rate their health worse, even after controlling for differences in education within counties (6.081, \( p < 0.01 \)).

\[
d_{30,ij} \sim N(XB, \Omega) \\
d_{30,ij} = \beta_{0ij}\text{cons} + \beta_{1ij}(\text{educ}_\text{imp-m(county}_\text{imp}))_{ij} + \\
6.081(1.019)(\text{unemrate-gm})_j
\]

\[
\beta_{0ij} = 2.634(0.015) + u_{0ij} + e_{0ij}
\]

\[
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\begin{bmatrix}
  1.113(0.007)
\end{bmatrix}
\]

\[\text{Cincinnati Children's} \]
Results and Discussion

• Variance/covariance components also need conditional interpretations.
  – After accounting for education’s and unemployment's effects, a relatively large amount of variance exists within counties (1.113).

\[
d_{30,ij} \sim N(X_\beta, \Omega)
\]
\[
d_{30,ij} = \beta_{0j} \text{cons} + \beta_{1j}(\text{educ}_\text{imp-m(county}_\text{imp}))_{ij} +
6.081(1.019)(\text{unemrate-gm})_{ij}
\]
\[
\beta_{0j} = 2.634(0.015) + u_{0j} + e_{0ij}
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\]
\[
\begin{bmatrix}
e_{0ij}
\end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix}1.113(0.007)\end{bmatrix}
\]
Results and Discussion

• Variance/covariance components also need conditional interpretations.
  – Controlling for education and unemployment, mean health still varies across counties (0.016).

\[
d30_{ij} \sim N(XB, \Omega)
\]
\[
d30_{ij} = \beta_0 + \beta_1 (\text{educ}_i - \text{m(county}_i)) + 6.081(1.019)(\text{unemrate}_i - \text{gm}_i)
\]
\[
\beta_0 = 2.634(0.015) + u_0j + e_{0j}
\]
\[
\beta_1 = -0.190(0.005) + u_1j
\]

\[
\begin{bmatrix}
  u_{0j} \\
  u_{1j}
\end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = 
\begin{bmatrix}
  0.016(0.003) \\
  -0.003(0.001) & 0.001(0.000)
\end{bmatrix}
\]

\[
\begin{bmatrix}
  e_{0ij}
\end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = 
\begin{bmatrix}
  1.113(0.007)
\end{bmatrix}
\]
Results and Discussion

- Variance/covariance components also need conditional interpretations.
  - Controlling for unemployment across counties, effect of education on health still varies across counties (0.001).

\[
\begin{align*}
  d_{30ij} & \sim N(XB, \Omega) \\
  d_{30ij} & = \beta_{0ij}\text{cons} + \beta_{1ij}(\text{educ}_\text{imp}-\text{m(county}_\text{imp}))_i + \\
  & \quad 6.081(1.019)(\text{unemrate-gm})_j \\
  \beta_{0ij} & = 2.634(0.015) + u_{0ij} + e_{0ij} \\
  \beta_{1ij} & = -0.190(0.005) + u_{1ij} \\
  \begin{bmatrix} u_{0ij} \\ u_{1ij} \end{bmatrix} & \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 0.016(0.003) \\ -0.003(0.001) 0.001(0.000) \end{bmatrix} \\
  \begin{bmatrix} e_{0ij} \end{bmatrix} & \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 1.113(0.007) \end{bmatrix}
\end{align*}
\]
Results and Discussion

- Variance/covariance components also need conditional interpretations.
  - Finally, even after controlling for unemployment, effect of education on health varies as a function of counties’ means (-0.003, correlation = -0.673).
  - In counties where individuals rate their health worse, low education has a more detrimental effect on health relative to counties where individuals rate their health better, even after controlling for differences in unemployment across counties.
Levels-1 and -2 with Interaction

• Model with Level 1 and Level 2 predictors and a cross level interaction.

• Addresses ALL of the previous questions and adds additional question......
  – Does the relationship between education and health become stronger (or weaker) in counties with more unemployment.
Results and Discussion

- Estimate describing interaction between education and health appears negative (-0.871: \( p < 0.01 \)).
  - In counties with more unemployment, low education particularly detrimental to health.

  - Remaining parameters retain similar conditional interpretations as in previous model, with condition that the model now includes the interaction term.

\[
d30_{ij} \sim N(XB, \Omega)
\]
\[
d30_{ij} = \beta_{0ij}\text{cons} + \beta_{1j}(\text{educ}_\text{imp}-m(\text{county}_\text{imp})) + 7.119(1.132)(\text{unemrate-gm}) + 0.871(0.416)(\text{educ}_\text{imp}-m(\text{county}_\text{imp}))1.(\text{unemrate-gm})_{ij}
\]
\[
\beta_{0ij} = 2.632(0.015) + u_{0j} + e_{0ij}
\]
\[
\beta_{1j} = -0.189(0.005) + u_{1j}
\]
Categorical Outcomes

• Suppose one seeks to model categorical outcome.
  – e.g., Does a child have access to a medical home or not?

• Increases in complexity and interpretation.

• Generally, use logistic regression to predict categorical outcome.
  – But, difficult to examine and interpret variance components in MLMs with categorical outcomes.
    – Transpires partly because of nonlinear relationship between covariates and outcome.
    – And difficulty partitioning variance.
Categorical Outcomes

• No clear distinction between individual and cluster-level variance.
  – If we know the prevalence of the outcome in each cluster, we know the variance within a cluster.
    • Not true with a continuous variable.
      – If we know cluster’s mean, we cannot infer cluster’s variance.

• Thus, cannot simply partition the variance like we do in a continuous model.
  – Yet, need remains to quantify variance across clusters and interpret in line with odds ratio interpretations.
    • See Merlo, et al. (2006) for more discussion.

• What can we do?
Median Odds Ratio (MOR)

- Quantifies variance among clusters.
  - Essentially compares two randomly chosen individuals with the same values on all covariates, but from two different clusters (i.e., contexts).
    - Conceptually, repeat this for all possible pairs.
      - Take the median odds ratio from all of these comparisons.
  - MOR gives the median odds ratio between individuals with higher propensity compared to people with lower propensity.
    - Always greater than or equal to .
    - If equal to 1, no variation exists between contexts.
    - If large, considerable variation exists.
    - Directly comparable to fixed-effects odds ratios.
      - Can make relative statements.
Median Odds Ratio (MOR)

- MOR summarizes variance across contexts among people with the same values on the covariates.
  - It encapsulates the increased risk that would occur if an individual moved from one context to another.
    - (In the median).
  - With no covariates in model, describes extent to which outcome depends on context.

- However, likely still wish to examine whether a contextual variable has large effect relative to unexplained variation between contexts.
Interval Odds Ratio (IOR)

- Quantifies the effect of contextual variables relative to variance across clusters.
  - How does the odds ratio for the contextual variable compare to the amount of variance across contexts after accounting for the contextual variable?
Interval Odds Ratio (IOR)

- Consider two random individuals with different values of a cluster-level covariate but same individual-level covariate values.
  - Compute odds for individual in context with higher propensity vs. lower propensity.
  - Consider all possible pairs of individuals.
    - Results in distribution of odds ratios (ORs).

- IOR = interval that contains 80% of these values.
  - If IOR contains 1, contextual variability large compared to effect of cluster-level variable.
  - If IOR does not contain 1, large cluster-level variable effect compared to unexplained contextual variation.
"A bushel is a unit of weight equal to four pecks.

What's a peck?

A quick smooch.

You know, I don't understand math at all.
Brief Example

• To what extent do individual- and state-level variables predict children’s access to a medical home?

• To what extent does variation exist across states?
Methods

• Used data from the 2007 National Survey of Children’s Health (NSCH).
  – Children \( n = 87,963 \) clustered within states.
  – Stratified, list-assisted random digit dial survey.
    • Stratified by state.
Methods

• For simplicity’s sake:
  – Uses unweighted data.
    • Weighted data \texttt{REQUIRE}s special techniques.
      – See resources.
      – Carle (2009).
  – Uses complete cases only.
    • Subpopping \texttt{REQUIRES} special techniques.
      – See resources.
Methods

- MLMs examined whether a child had access to a medical home as operationalized in NSCH.

- Predicted as a function of:
  - Race and ethnicity.
    - Non Hispanic (NH) White.
    - NH Black.
    - NH Other.
    - Hispanic.
  - Level-2 predictor:
    - % children in state with gaps in insurance coverage.

- Manuscript in progress.
  - Please do not replicate and publish (yet).
Models

• Unconditional model.
  – Examines whether access to medical homes varies across states.

• Level-1 predictor only.
  – Does race/ethnicity predict access to a medical home?
    • Does relationship between race/ethnicity and access vary across contexts?

• Level-1 and Level-2 predictor model.
  – Do race/ethnicity and % of children in state with insurance gaps predict access to a medical home?
Results and Discussion

• Unconditional model.
  – Notice no longer have variance component for “within.”
  – Unadjusted odds of having access to a medical home:
    • \( \text{OR} = \exp(0.485) = 1.624 \).
Results and Discussion

• Unconditional model.
  – MOR = \( \exp\left(\sqrt{2 \times 0.047 \times 0.675}\right) = 1.231 \)
  – If child moved to state with higher probability of access to medical home, likelihood they would have access to medical home would increase by 1.23.

• Nearly 25% more likely to access medical home if moving to state with higher likelihood of access to medical home.
Level 1 Model

- Model with only a Level 1 predictor.
  - Does race/ethnicity predict medical home access and does that relationship differ across states?
Results and Discussion

- Level 1 predictor with fixed and random effects.
  
  \[ \text{MOR} = \exp\left(\frac{\sqrt{2}}{0.014} \times 0.675\right) = 1.119 \]

\[
\begin{align*}
\text{med}_{ij} & \sim \text{Binomial}\left(\text{denom}_{ij}, \pi_{ij}\right) \\
\text{logit}(\pi_{ij}) &= \beta_{0j}\text{cons} + \beta_{1j}\text{African American, non-Hispanic}_{ij} + \beta_{2j}\text{Hispanic}_{ij} + \beta_{3j}\text{Multi/Other, non-Hispanic}_{ij} \\
\beta_{0j} &= 0.765(0.019) + u_{0j} \\
\beta_{1j} &= -0.880(0.027) + u_{1j} \\
\beta_{2j} &= -0.985(0.039) + u_{2j} \\
\beta_{3j} &= -0.553(0.039) + u_{3j}
\end{align*}
\]

\[
\begin{bmatrix}
u_{0j} \\
u_{1j} \\
u_{2j} \\
u_{3j}
\end{bmatrix} \sim \text{N}(0, \, \Omega_u) : \Omega_u = \\
\begin{bmatrix}
0.014(0.003) & 0 & 0.004(0.006) \\
0 & 0.042(0.014) & 0 \\
0 & 0 & 0.041(0.015)
\end{bmatrix}
\]

\[\text{var}(\text{med}_{ij} | \pi_{ij}) = \pi_{ij}(1 - \pi_{ij}) / \text{denom}_{ij} \]
Results and Discussion

• Level 1 predictor with fixed and random effects.
  – \[ \text{MOR} = \exp\left(\sqrt{2}(0.014)\right)\left(0.675\right) = 1.119 \]

  – After controlling for differences attributable to differential distribution of race and ethnicity within states, odds of accessing a medical home in higher compared to lower propensity state now 1.12.

• Differences in distribution of race/ethnicity across states explained some variance in accessing medical home across states.

• Variation due to context (MOR=1.12) less relevant than impact of a child’s race/ethnicity (OR\_range: 0.373-0.575).

• Some but not all variance explained.
Level-1 and Level-2 Model

- Model with Level 1 and Level 2 predictors.
  - Examine relationship between % of children with insurance gaps and access to a medical home after controlling for within state race/ethnicity differences.
  - Does the relationship between access and race/ethnicity differ across states after controlling for differences in % of children with insurance gaps?
Results and Discussion

- Level 1 and level 2 predictors.
  - \( \text{MOR} = \exp(\left(\sqrt{(2)(0.006)}(0.675)\right)) = 1.077 \)

\[
\begin{align*}
\text{med}_{\text{home}_y} & \sim \text{Binomial}(\text{denom}_y, \pi_y) \\
\text{logit}(\pi_y) &= \beta_0\text{cons} + \beta_1\text{African American, non-Hispanic}_{ij} + \beta_2\text{Hispanic}_{ij} + \beta_3\text{Multi/Other, non-Hispanic}_{ij} + \beta_4\text{sgapins-gm}_{ij} \\
\beta_0 &= 0.764(0.014) + u_{0j} \\
\beta_1 &= -0.877(0.027) + u_{1j} \\
\beta_2 &= -0.981(0.038) + u_{2j} \\
\beta_3 &= -0.553(0.039) + u_{3j} \\

\begin{bmatrix}
u_{0j} \\
u_{1j} \\
u_{2j} \\
u_{3j}
\end{bmatrix} & \sim \mathcal{N}(0, \Omega_u) : \Omega_u = \\
\begin{bmatrix}
0 & 0.006(0.002) \\
0.006(0.002) & 0.004(0.005) \\
0 & 0 & 0.040(0.014) \\
0 & 0 & 0 & 0.040(0.015)
\end{bmatrix}
\end{align*}
\]

\[\text{var}(\text{med}_{\text{home}_y}|\pi_y) = \pi_y(1 - \pi_y)/\text{denom}_y\]
Results and Discussion

• Level 1 and level 2 predictors.
  – \[ \text{MOR} = \exp\left(\sqrt{2} \times 0.006 \times 0.675\right) = 1.077 \]

  – After controlling for differences attributable to differential distribution of race and ethnicity within states and % of children with insurance gaps, odds of accessing a medical home in higher compared to lower propensity state now 1.077.

• % of children with gaps did not have a large impact on variation across states.
  – Change from 1.119 to 1.077.
Results and Discussion

- **Level 1 and level 2 predictors.**
  - $\text{IOR}_{\text{lower}} = \exp\left(0.028\left(\sqrt{2}\right)(0.006)(-1.28)\right) = 0.794$
  - $\text{IOR}_{\text{upper}} = \exp\left(0.028\left(\sqrt{2}\right)(0.006)(1.28)\right) = 1.052$

$\text{med\_home}_y \sim \text{Binomial}(\text{denom}_y, \pi_y)$

$logit(\pi_y) = \beta_0\text{cons} + \beta_1\text{African American, non-Hispanic}_y + \beta_2\text{Hispanic}_y + \beta_3\text{Multi/Other, non-Hispanic}_y + -0.090(0.014)\text{(sgapins-gm)}_y$

$\beta_0 = 0.764(0.014) + u_{0j}$
$\beta_1 = -0.877(0.027) + u_{1j}$
$\beta_2 = -0.981(0.038) + u_{2j}$
$\beta_3 = -0.553(0.039) + u_{3j}$

$\begin{bmatrix}
u_{0j} \\
u_{1j} \\
u_{2j} \\
u_{3j}
\end{bmatrix} \sim N(0, \Omega_u) : \Omega_u =
\begin{bmatrix}
0.006(0.002) & 0 & 0 & 0.040(0.014) \\
0 & 0.004(0.005) & 0 & 0 \\
0 & 0 & 0 & 0.040(0.015)
\end{bmatrix}$

$\text{var} (\text{med\_home}_y | \pi_y) = \pi_y (1 - \pi_y) / \text{denom}_y$
Results and Discussion

• $\text{IOR}_{80} = 0.794 - 1.052$.
  – Recall, children residing in states where more children had insurance gaps had lower odds of accessing a medical home compared to children in states where fewer children had gaps:
  • $\text{OR} = \exp(-.09) = 0.914$.
    – Controlling for differences in race/ethnicity within states.
Results and Discussion

• $\text{IOR}_{80} = 0.794 - 1.052$.
  – However, IOR relatively wide.
  • Indicates relatively large between state variation.
    – Randomly choose two children with same covariates, but one from state with large % gaps and one with low % gaps, OR lies with $0.794 - 1.052$ 80% of the time.
    – % with gaps does not explain large proportion of variance across states.

• Also indicates small probability exists that a child from a state with large % of gaps may still have a greater likelihood of accessing a medical home.
  – “Small” because most of the interval ranges below 1.

• Other state level variables needed to explain state heterogeneity.
Conclusion

• Power of MLM lies in their ability to examine what predicts variance across contexts. As well as include individual and contextual level variables.

• What predicts differences across contexts? MLM allow us to empirically explore this.

• Correctly incorporating MLM into epidemiology will advance our understanding of all influences on people’s health.
Resources

• General Resources:
  
  
  
Resources

• General Resources:
Resources

• Online Resources:
  – UCLA’s statistics site.
    • http://www.ats.ucla.edu/stat/

  – Center for Multilevel Modeling (MLwiN).
    • http://www.cmm.bristol.ac.uk/links/index.shtml

  – Judith Singer’s site (deals mainly with longitudinal models, but still very useful).
    • http://gseacademic.harvard.edu/~alda/

  – Scientific Software International’s site.
    • http://www.ssiscentral.com/hlm/references.html#r7
Resources

• Centering:


Resources

• Software:
  – SPSS:

  – GLLAMM:
Resources

• Software:
  – Stata:
    • Rabe-Hesketh S, Skrondal A. Multilevel and Longitudinal Modeling using Stata. College Station: Stata; 2005.
  
  – SAS:
Resources

• Software:
  – HLM:
  – Mplus:
Resources

• Categorical MLM:

Resources

• Categorical MLM:

Resources

• Categorical MLM:
Resources

- MLM with Complex Survey and Design Weights:
  - Carle AC. Fitting multilevel models in complex survey data with design weights: Recommendations. BMC Medical Research Methodology. 2009;9(49).
Understanding and Applying Multilevel Models in Maternal and Child Health Epidemiology and Public Health

Adam C. Carle, M.A., Ph.D.
adam.carle@cchmc.org
Division of Health Policy and Clinical Effectiveness
James M. Anderson Center for Health Systems Excellence
Cincinnati Children’s Hospital Medical Center
University of Cincinnati School of Medicine
Cincinnati, OH

16th Annual Maternal and Child Health Epidemiology Conference
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